agreement with the available experimental data and lend confidence to CFD but comparing the different results was difficult due to the quality of the converged solutions. Other questions arose concerning symmetry with several authors assuming this to be the case from the start of the computation. Some authors attempting the full cavity problem observed a symmetric development of the flow whilst others detected a loss of symmetry. The issue of grid resolution and time step was also investigated and their effect on the development of the Taylor-Görtler-like vortices clearly demonstrated. The second test case, concerning the steady state flow around a prolate spheroid at incidence, produced similar results from the two groups involved. The book presents a timely assessment of current methodologies involved in CFD applied to 3-D problems and lays the foundations for the development of more accurate reference solutions for future benchmark problems. It clearly illustrates the need for further work in this field and paves the way for the more complex issue of transient, turbulent flow simulation.

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Box Splines
C. de Boor, K. Höllig, S. Riemenschneider
Applied Mathematical Sciences, vol. 98
Springer–Verlag, Berlin, 1993, XVII + 200 pages, 57 figs.
Price: DM 60 (HC)
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This book carefully exposes the intriguing and beautiful mathematical theory of box splines and box spline spaces and shows how these are, in many respect, the multivariate equivalent of the univariate cardinal splines studied intensively by Schoenberg and others.
Loosely stated, a box spline $M_\Xi(x)$, $x \in \mathbb{R}^s$, is a smooth piecewise polynomial function with local support that can be associated with a matrix $\Xi \in \mathbb{R}^{s\times n}$. The number of columns n and other properties of Ξ, such as its rank, determine the degree of the polynomials and the order of continuity of M_Ξ. The support of the box spline is given by $\{\sum t_i c_i : 0 \le t_i \le 1\}$ in which the c_i denote the different columns of Ξ. Well known examples are the uniform univariate B-spline having knots $\{0, 1, \ldots, n\}$ which is associated with $\Xi = [1\ 1\ \cdots 1] \in \mathbb{R}^{1\times n}$ and the corresponding multivariate tensor product B-splines in which all the c_i are columns of the s–dimensional identity matrix. Other important bivariate box splines are those for which Ξ consists of the column vectors $i_1 = [1\ 0]^T$, $i_2 = [0\ 1]^T$ and $i_3 = i_1 + i_2$ (and $i_4 = i_2 - i_1$), each with a multiplicity ≥ 1, which are termed as box splines on the three– (respectively four–) direction mesh. These and other specific examples are reconsidered throughout the book to illustrate new symbols, definitions and theorems.
A more rigorous definition of the box spline is given in Chapter 1 of the book. In fact the authors present no less than three alternative and equivalent definitions. Subsequently they derive the basic properties of M_Ξ (recurrence relation, derivatives, Fourier transform, polynomial degree and order of continuity, support and cut regions, ...). A single box spline is of little practical use. In the further chapters Ξ is restricted to have integer entries and shifts of such a box spline are then considered, i.e. $M_\Xi(x - j)$, $j \in \mathbb{Z}^s$. The linear algebra of a cardinal spline space $S := \{\sum a(j) M_\Xi(x - j)\ ;\ a(j) \in \mathbb{R}\}$ is the topic of Chapter 2. It highlights the results of Dahmen and Micchelli on linear independence of box spline shifts and also addresses the question what polynomials are contained in S. This last question is connected with the approximation order of S which is further discussed in Chapter 3. This chapter also includes a discussion on the construction of quasi–interpolants which realize that order. In Chapter 4 cardinal interpolation is considered, i.e. the problem of finding the cardinal spline $\in S$ whose coefficients $a(j)$ satisfy the difference equation $\sum a(j) M_\Xi(k - j) = f(k)$, $k \in \mathbb{Z}^s$. A basic question to be answered in this respect is whether the problem is *correct* or *singular*, i.e. whether for any bounded f there exists a unique bounded solution a or, on the other hand, whether there is a non–trivial solution a in case $f \equiv 0$. Chapter 5 studies the approximation by cardinal spline series. In the first part convergence results are obtained in case the degree tends to infinity (a sequence of matrices Ξ_r is considered consisting of r copies of Ξ). In the second part Ξ is fixed while through

an appropriate scaling the mesh width now goes to zero. The theory of *discrete* box splines is developed in Chapter 6, in close analogy to that of the (*continuous*) box spline in Chapter 1. It provides the basis for the discussion of subdivision algorithms in the final chapter. Such an algorithm finds the representation of a given cardinal box spline on a finer lattice, i.e. it computes the coefficients $b(k)$ from $a(j)$ such that $s(x) = \sum a(j)M_{\Xi}(x-j) = \sum b(k)M_{\Xi}((x-k)/h)$, $h \in 1/I\!N$. Convergence results are obtained for repeated subdivision.

The authors have succeeded in presenting the available basic material on box splines in a cohesive and also very didactical way. Illustrative are the many detailed examples and well chosen figures which certainly help to digest this hearty piece of theory. Assuming that most readers are familiar with the first author's standard work on splines – vol. 27 in the same series – some warning seams appropriate, however. This book is not intended to be "A Practical Guide to Splines, Part II". The mathematics involved are far more advanced and very little attention is paid to the computational aspects of box splines. Their implementation in finite element packages or CAD systems, as the authors suggest in their preface, therefore seems a non-trivial extrapolation. However, it doesn't detract from the many merits of this book, which will surely charm the mathematician interested in Approximation Theory.

P. Dierckx

2 Announcements of conferences

IFIP WG10.3 Working Conference on PROGRAMMING ENVIRONMENTS FOR MASSIVELY PARALLEL DISTRIBUTED SYSTEMS

Date: 25–30 April, 1994.

Location: Monte Verita, Ascona, Switzerland.

Other information: CAM-Newsletter 9, nr. 3.

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SPECTRAL MULTI-DOMAIN METHODS WORKSHOP

Date: 16–18 May, 1994.

Location: North Carolina State Un., Raleigh, U.S.A.

Other information: CAM-Newsletter 9, nr. 3.

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International Spring School NONLINEAR ANALYSIS, FUNCTION SPACES AND APPLICATIONS V

Date: 22-28 May, 1994.

Location: Prague, Czech Republic.

Organizers:
- The Institute of Mathematics, Czech Academy Sciences in Prague.
- The University of West Bohemia in Pilsen.

Lectures:
- F. Chiarenza (Catania): L^p-regularity for systems of PDE's, with coefficients in VMO.
- D.E. Edmunds (Sussex): Recent developments concerning entropy and approximation numbers.
- B. Kawohl (Köln): On the shape of solutions to some variational problems.
- F.J. Martin-Reyes (Malaga): One-sided operators, weights and applications.